PRACTICAL LESSON No. 1
TOPIC: ELEMENTS OF THE THEORY OF PROBABILITIES

OBJECTIVE
1. To study the basic concepts of the theory of probabilities;
2. Learn to solve problems for finding the probabilities of random events;
3. To study the laws of distribution of random variables and their main characteristics: mathematical expectation and variance.

The student must study the material on the topic and be able to answer the following questions:
1. The concept of evidence-based medicine.
2. A random event. The probability of a random event. Classical and statistical definition of probability.
3. The concept of joint (compatible) and incompatible events. The law (theorem) of addition of probabilities. Examples of compatible and incompatible events.
4. The concept of dependent and independent events. Conditional probability, the law (theorem) of multiplication of probabilities. Examples of dependent and independent events.
5. Continuous and discrete random variables.
6. Distribution of discrete and continuous random variables and their characteristics: mathematical expectation, variance, standard deviation.
7. Distribution function. Probability density.
8. Normal distribution law of continuous random variables. Exponential law of distribution of continuous random variables.

1. RANDOM EVENTS

Theory of probabilities is a branch of mathematics that studies random events, random variables, their properties and operations on them.
Probability theory studies probabilistic patterns of mass homogeneous random events.

Mathematical statistics is a branch of mathematics that studies the methods of collecting, systemizing, processing and using statistical data to obtain scientifically based conclusions and make decisions based on them.

A **random event** is an event that may either occur or may not occur as a result of a test (experiment). A random event is denoted by a capital Latin letter *A*, *B*, *C*, *D*…

Test (experiment) – that is a certain complex (set) of conditions which can be reproduced.
Example 1. The appearance of a head (or a tail) when a coin is thrown is a random event.

Example 2. Hitting a given object or a given area when firing at this object from a given weapon is a random event.

Two or more events are called **equally possible** if none of them is more possible than the other(s).
Example 1. Appearance of a head or a tail when throwing up coins;
Example 2. Appearance of 1, 2, 3, 4, 5 or 6 points when tossing a die;
Example 3. Extraction of diamonds, clubs, spades or hearts card from a deck.

An event is said to be **certain** if it necessarily occurs in the result of a test.

An event is called **impossible** if it never occurs in the result of a test.

Events are called **incompatible** if the appearance of one of them excludes the appearance of the other.
(Random events are said to be incompatible if no two of them can appear together in a given test).

Events are called **compatible** (joint) if the appearance of one of them does not exclude the appearance of the other.

Two events *A* and $\bar{A}$ (not *A*) are called **contrary** if nonoccurrence of one of them leads to occurrence of the other.

Random events form a **complete group** if, at each trial, any of them can appear and any two of them are incompatible.
(The complete group of events is such a system of random events that, as a result of a random experiment, one and only one of them will certainly occur).

Let us consider a **complete group of equally possible incompatible random events**. Such events are called **outcomes** or **elementary event**s.
The outcome is said to **favor** the occurrence of event *A* (is **favorable** to the event *A*), if the appearance of this outcome entails the occurrence of event *A*.
Example. There are 8 numbered balls in the urn (one number from 1 to 8 is placed on each ball). Balls with numbers 1, 2, 3 are red, the rest are black. The appearance of a ball with the number 1 (or the number 2 or the number 3) is an event favorable to the appearance of the red ball. The appearance of a ball with the number 4 (or the number 5, 6, 7, 8) is an event favorable to the appearance of a black ball.

The **probability of an event** *A* is the ratio of the number *m* of outcomes favorable to this event to the total number *n* of all equally possible incompatible elementary outcomes forming the full group:

$$P(A) = \frac{m}{n}$$

So, the probability of a random event is the numerical degree of objective possibility of an event occurrence.
Property 1. The probability of certain event is equal to one.
Property 2. The probability of an impossible event is zero.
Property 3. The probability of a random event is a positive number, enclosed between zero and one:

$$0 \le P(A) \le 1.$$

Problem 1.1. There are 15 balls in the urn: 5 white and 10 black. What is the probability to remove the blue ball from the urn?
Solution. Since there are no blue balls in the urn, then m = 0, n = 15. Therefore, the desired probability is P(blue) = 0. The event "taking out the blue ball" is impossible.

Problem1.2. From the deck of 36 cards, one card is removed. What is the probability that this card is of hearts suit?
Solution:
The number of elementary outcomes (number of cards) n = 36.
Event A - the appearance of the card of hearts suit. The number of cases favoring the occurrence of event A, m = 9.
Consequently, $P(\mathrm{A}) = \frac{m}{n} = \frac{9}{36} = 0{,}25$

Problem 1.3. A laboratory rat placed in a maze must choose one of five possible paths. Only one of these leads to food rewards. Assuming that any choice is equiprobable, what is the probability of the chosen path to lead to food?
Solution: $P(A)$ = 1/5

Problem 1.4. When throwing a die, six outcomes are probable: 1, 2, 3, 4, 5, 6 points. What is the probability of having an even number of points?
Solution: three outcomes (out of six possible) - 2, 4 and 6 points – are favorable to the event A: "Even number of points appear"; $P(A)$ = 3/6 = 1/2.

Problem 1.5. All natural numbers from 1 to 30 are written on identical cards and placed in the urn. After thorough mixing of the cards, one card is removed from the urn. What is the probability that the number on the taken card will be a multiple of 5?
Solution: Let A is the event consisting in the fact that the number of the card taken is a multiple of 5. In this test there are 30 equally possible outcomes, of which event A is favored by 6 outcomes (5, 10, 15, 20, 25, 30), therefore,
$P(A)$ = 6/30 = 1/5.

Problem 1.6. Two coins are tossed. What is the probability that both will fall with "heads" up?

Solution: Four outcomes of throwing two coins are possible: HH, HT, TH, TT. Let A is the event “two heads” (HH). Only one outcome is favorable to this event: $P(A) = 1/4$.

Problem 1.7. Two dice are tossed, the sum of points on the upper edges is calculated. Which is more likely to get: a total of 7 or 8?

Solution: Let the event A is: “7 points dropped out”; the event B - “8 points dropped out”.

Event A is favored by 6 elementary outcomes:

(1; 6), (2; 5), (3; 4), (4; 3), (5; 2), (6; 1).

Event B is favored by 5 outcomes: (2; 6), (3; 5), (4; 4), (5; 3), (6; 2).

The number of all equipossible outcomes $n = 6 \cdot 6 = 36$.

$$P(\mathrm{A}) = \frac{6}{36} = \frac{1}{6} = 0{,}167$$

$$P(\mathrm{B}) = \frac{5}{36} = 0{,}139\,.$$

So, P (A) > P (B): the probability to obtain 7 on upper verges is greater than to get 8.

The **relative frequency** (or simply the **frequency**) of a random event A is the ratio of the number of occurrences of this event to the total number of identical tests conducted, in each of which this event could appear or not:

$$P^*(A) = p^* = \frac{m^*}{n^*}$$

The relative frequency of an event is the proportion of those actually performed tests in which event A occurred. This is an experimental characteristic, where m^* is the number of experiments in which the event A appeared; n^* is the number of all experiments performed.

The probability of an event is the number around which the values of the frequency of a given event are grouped in various series of a large number of tests.

Example. 6 series of shots are made (event A - hitting the target) with a given gun under the same conditions:

in the 1-st series there were 5 shots, the number of hits was 2, relative frequency 2/5;

in the 2-nd series there were 10 shots, the number of hits was 6, relative frequency 6/10;

in the 3-rd series there were 12 shots, the number of hits 7, relative frequency 7/12;

in the 4-th series there were 50 shots, the number of hits was 27, relative frequency 27/50;

in the 5-th series there were 100 shots, the number of hits was 49, relative frequency 49/100;

in the 6-th series 200 shots, the number of hits 102, relative frequency 102/200.

Experience shows that in the overwhelming majority of cases there is a constant number *p* such that the relative frequencies of occurrence of an event *A* at a large number of tests, except in rare cases, differ little from this number *p*:

$$\frac{m^*}{n^*} \underset{n^* \to \infty}{\longrightarrow} p$$

$$\lim_{n^* \to \infty} \frac{m^*}{n^*} = P(A)$$

The formula represents the statistical definition of probability of a random event.

Problem 1.8. Of the 982 patients admitted to the surgical hospital in a month, 275 people had injuries. What is the relative frequency of admission of patients with this type of disease?

Solution: $\frac{m^*}{n^*} = \frac{275}{982}$

Problem 1.9. When shooting at a target, the hit rate (relative frequency of hits) w = 0,75. Find the number of hits for 40 shots.

Solution: $w = \frac{m^*}{n^*};\ m^* = w \cdot n^* = 0{,}75 \cdot 40 = 30$

THE LAW OF ADDITION OF PROBABILITIES

The **sum of two events** is an event in which at least one of these events (*A* or *B*) appears.
If *A* and *B* are compatible (joint) events, then their sum (*A* + *B*) denotes the occurrence of event *A* or event *B* or both events together.
If *A* and *B* are incompatible events, then the sum of (*A* + *B*) means the occurrence of event *A* or event *B*.

Problem 1.10. The winner of the competition is awarded a diploma (event A), a cash prize (event B), a medal (event C). What is the event (A + B)?
Solution: Event (A + B) consists of awarding the winner with either a diploma or a cash prize, or both.

Problem 1.11. The tourist has the opportunity to visit 3 cities. Event designations:
A - the tourist has visited the city A;
B - the tourist has visited the city B;
C - the tourist has visited the city C.
What is the event A + C?
Solution: The tourist has visited either city A, or city C, or both of them.

The **probability of the sum of incompatible events** is equal to the sum of the probabilities of these events:

$$P(A + B) = P(A) + P(B).$$

The probability of the **sum of two compatible (joint) events** is equal to the sum of the probabilities of these events without the probability of their joint occurrence:

$$P(A + B) = P(A) + (B) - P(AB).$$

The sum of the probabilities of discrete events that form a complete group is equal to 1:

$$P(A_1) + P(A_2) + \ldots + P(A_n) = 1$$

or

$$\sum_{i=1}^{n} P(A_i) = 1$$

The sum of the probabilities of opposite events is equal to 1:

$$P(\mathrm{A}) + P(\overline{\mathrm{A}}) = 1$$

Problem 1.12. Let the probability that Jim wins the race is 1/3 and the probability that Tom wins the race is 1/5. What is the probability that one of them will win the race?

Solution: $P(\mathrm{A}+\mathrm{B}) = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$.

Problem 1.13. In the money and clothing lottery, 150 items and 50 money prizes are drawn for every 10,000 tickets. What is the probability of winning, it does not matter, money or clothing, for the owner of one lottery ticket.

Solution:

$$P(\mathrm{A}+\mathrm{B}) = \frac{150}{10000} + \frac{50}{10000} = 0{,}02\,.$$

Problem 1.14. The probability of hitting the tumor cell ("target") for the first radionuclide is P_1 = 0.7, and for the second - P_2 = 0.8. Find the probability of hitting the “target” if both drugs were used simultaneously.

Solution 1: $P(A+B) = P(A) + P(B) - P(A \cdot B) = 0{,}7 + 0{,}8 - 0{,}56 = 0{,}94$.

Solution 2:

$$P(\overline{\mathrm{A}}) = 1 - P(A) = 1 - 0{,}7 = 0{,}3$$

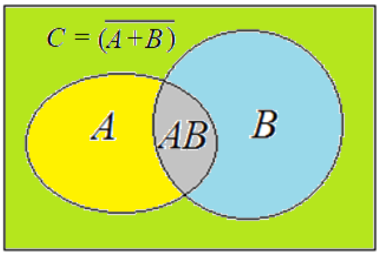
$$P(\overline{B}) = 1 - P(B) = 1 - 0{,}8 = 0{,}2$$

$$P(\overline{A} \cdot \overline{B}) = P(\overline{A}) \cdot P(\overline{B}) = 0{,}3 \cdot 0{,}2 = 0{,}06$$

$$P(\text{of hitting the target}) = P(A + B) = 1 - P(\overline{A} \cdot \overline{B}) = 1 - 0{,}06 = 0{,}94$$

Problem 1.15. In a large fruit fly population, 25% of flies have an eye mutation, 50% have a wing mutation, and 40% of flies with an eye mutation also have a wing mutation. What is the probability that a fly chosen at random from this population will have at least one of these mutations? What is the probability that a randomly chosen fly has no mutations?

Solution:



A - an event that a randomly selected fly has eye mutations, $P(A)$ = 0,25.
B - an event that a randomly selected fly has a wing mutation, $P(B)$ = 0,5.
AB – an event that a randomly selected fly has both mutations, $P(A \cdot B) = 0{,}4 \cdot P(A)$.
C - an event that a randomly selected fly has no mutations.
The probability that a fly has one or both mutations:

$$P(A+B) = P(A) + P(B) - P(A \cdot B) = 0{,}25 + 0{,}5 - 0{,}4 \cdot 0{,}25 = 0{,}65.$$

The probability that a randomly chosen fly has no mutations

$$P(C) = 1 - P(A+B) = 1 - 0{,}65 = 0{,}35$$

CONDITIONAL PROBABILITY

The conditional probability $P(B/A)$ of event B is the probability of event B, found under the condition that event A has occurred.

Problem 1.16. The box contains 3 white and 3 yellow pills. One pill is taken out of the box at random twice, without returning them to the box. Find the probability of white pill appearing in the second trial (event B) if a yellow pill was removed in the first trial (event A).
Solution: After the first test, there are 5 tablets left in the box, of which 3 are white.The required conditional probability: $P(B/A) = 3/5 = 0{,}6$

Problem 1.17. The box contains 8 red and 6 white pills. 3 pills are taken out of the box sequentially without returning. Find the probability that all 3 pills are white.
Solution:
A_1 - the first tablet is white;
A_2 - the second tablet is white;
A_3 - the third tablet is white.

$$P(A_1A_2A_3) = P(A_1) \cdot P(A_2/A_1) \cdot P(A_3/A_2/A_1)$$

$P(A_1) = 6/14$

$P(A_2/A_1) = 5/13$

$P(A_3/A_2/A_1) = 4/12$

$$P(A_1A_2A_3) = P(A_1) \cdot P(A_2/A_1) \cdot P(A_3/A_2/A_1) = \frac{6}{14} \cdot \frac{5}{13} \cdot \frac{4}{12} = 0{,}055$$

THE LAW OF MULTIPLICATION OF PROBABILITIES

The **product of two events is an event consisting in the joint appearance** of these events (*A* and *B*).

Problem 1.18. Let there be the following events: A - “a queen is taken out of the deck of cards”; B - “a card of spades is drawn from the deck of cards”. What is an AB event?
Solution:
AB is the event “the queen of spades is taken out”.

Event *B* is said to be **independent** of event *A* if the occurrence of event *A* does not change the probability of occurrence of event *B*.
The probability of the occurrence of several independent events is equal to the product of the probabilities of these events:

$$P\,(A \cdot B) = P\,(A) \cdot P(B).$$

For **dependent** events:
$P(AB) = P(A) \cdot P(B/A)$.
The probability of the product of two events is equal to the product of the probability of one of them by the conditional probability of the other, found under the assumption that the first event occurred.

Problem 1.19. Find the probability of the joint appearance of heads with one toss of two coins.

Solution: $P(AB) = P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Problem 1.20. Two dice are tossed. What is the probability that the first die will have an even number of points, and the second will have a number less than 6?

Solution: $P(AB) = P(A) \cdot P(B) = \frac{3}{6} \cdot \frac{5}{6} = \frac{5}{12}$

Problem 1.21. Find the probability that in families of two children 1) both children are boys; 2) both children are girls; 3) the eldest child is a boy, and the youngest is a girl. The probability of having a boy is 0.515.

Solution: $P(BB) = P(B) \cdot P(B) = 0{,}515 \cdot 0{,}515 = 0{,}265$

$P(GG) = P(G) \cdot P(G) = 0{,}485 \cdot 0{,}485 = 0{,}235$

$P(BG) = P(B) \cdot P(G) = 0{,}515 \cdot 0{,}485 = 0{,}25$

Problem 1.22. The probability that a student will pass the first exam during the summer session is 0.9; the second - 0.9; third - 0.8. Find the probability that the student will pass:
1) only the 2nd exam

2) all the 3 exams.
Solution: event A – first exam passed; event B – second exam passed; event C – third exam passed.

1)
$$P(\overline{A}B\overline{C}) = P(\overline{A}) \cdot P(B) \cdot P(\overline{C}) = (1 - P(A)) \cdot P(B) \cdot (1 - P(C)) = 0{,}1 \cdot 0{,}9 \cdot 0{,}2 = 0{,}018$$
2). $P(ABC) = P(A) \cdot P(B) \cdot P(C) = 0{,}9 \cdot 0{,}9 \cdot 0{,}8 = 0{,}648$

Problem 1.23. The probability of hitting the target when firing from three guns is as follows: $P(A) = 0{,}75$; $P(B) = 0{,}8$; $P(C) = 0{,}85$.
What is the probability of at least one hit with one salvo from all these guns?
Solution: $P(\overline{A}) = 0{,}25$; $P(\overline{B}) = 0{,}2$; $P(\overline{C}) = 0{,}15$.
$$P(\text{no hit at all}) = P(\overline{A}\overline{B}\overline{C}) = P(\overline{A})P(\overline{B})P(\overline{C}) = 0{,}25 \cdot 0{,}2 \cdot 0{,}15 = 0{,}0075$$
$$P(\text{at least one hit}) = 1 - P(\overline{A}\overline{B}\overline{C}) = 1 - 0{,}0075 = 0{,}9925$$

Problem 1.24. Two shooters shoot at the target. The probability of hitting the target with one shot for the first shooter is 0.7, and for the second - 0.8. Find the probability that the target is hit only by one bullet when each of them shoot once.
Solution:
The probability that the first shooter hits the target and the second does not hit is
$$P(A_1\overline{A}_2) = 0{,}7 \cdot (1 - 0{,}8) = 0{,}7 \cdot 0{,}2 = 0{,}14$$
The probability that the second shooter hits the target and the first does not hit it –
$$P(\overline{A}_1 A_2) = (1 - 0{,}7) \cdot 0{,}8 = 0{,}3 \cdot 0{,}8 = 0{,}24$$
The probability that only one shooter will hit the target is equal to the sum of these probabilities:
$$P(\text{only one hit}) = P(A_1\overline{A}_2) + (\overline{A}_1 A_2) = 0{,}14 + 0{,}24 = 0{,}38$$

Problem 1.25. How many children should a couple plan to have so that the probability of at least one boy is not less than 90% (the probability of having a boy and a girl is 0.5).
Solution: the probability that all the n children in the family are girls:
$$P(\text{all are girls}) = P(G) \cdot P(G) \cdot \ldots \cdot P(G) = \frac{1}{2} \cdot \frac{1}{2} \cdot \ldots \cdot \frac{1}{2} = \left(\frac{1}{2}\right)^n$$
The probability that not all girls are:
$$P(\text{at least one boy}) = 1 - P(\text{all are girls}) = 1 - \left(\frac{1}{2}\right)^n.$$
$$1 - \left(\frac{1}{2}\right)^n = 0{,}9$$
$$0{,}1 = \left(\frac{1}{2}\right)^n$$

$$\lg 0{,}1 = \lg\left(\frac{1}{2}\right)^n$$

$$n = \frac{-1}{\lg 0{,}5} \approx 3{,}3$$

Answer: the number of children must be greater than 3 (so, at least 4).

2. RANDOM VARIABLES

A **random variable** is a variable that may take one or another value, it is not known in advance what exactly, as a result of a test.

Discrete (discontinuous) random variable – that is a random variable that takes a countable number of values. (Possible values of discontinuous random variable can be pre-listed).
Examples: number of students at the lecture; number of newborn babies during a week etc.

Continuous random variable – that is a random variable possible values of which cannot be pre-listed and continuously fill a certain interval.
Examples: readings of a thermometer; humidity of air; the value of systolic pressure etc.

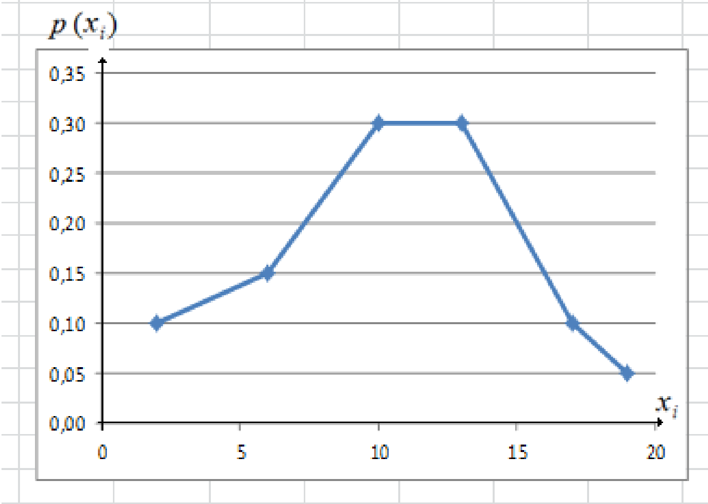
Random variables are denoted by capital Latin letters *X*, *Y*, *Z* and their possible values – by the corresponding small letters *x*, *y*, *z*.
Example: a random variable X - the number of hits with three shots; it can take 4 possible values:

$$x_1 = 0; x_2 = 1; x_3 = 2; x_4 = 3$$

The **law of distribution of a random variable** – that is any relationship that gives a correspondence between the possible values of a random variable and the probabilities of these values.

x_i – values of a random variable X	2	6	10	13	17	19
p_i – probability of x_i	0,10	0,15	0,30	0,30	0,10	0,05

Another way to present the law of distribution of a discrete random variable is a **polygon of probabilities distribution**.
For the above shown example it looks as shown in the picture.



Problem 2.1: There are 100 tickets issued in the cash lottery. There is one win of 50 rubles and 10 wins of 1 ruble each. Find the distribution law for a random variable X - the cost of a possible win.
Solution: Possible X values: 0, 1, 50.
Law of distribution:

x_i	0	1	50
p_i	0,89	0,1	0,01

Problem 2.2: The probability that a student passes a semester exam in biophysics (A_1) is 0.7, and for biochemistry (A_2) is 0.9. Construct the law of distribution for the number of semester exams that a student will pass.
Solution: Possible values of X - number of passed exams: 0, 1, 2.
The probabilities are:

$$P(\text{no exams passed}) = P(\bar{A}_1\bar{A}_2) = (1-0{,}7)\cdot(1-0{,}9) = 0{,}03$$

$$P(\text{only one exam}) = P(A_1\bar{A}_2) + (\bar{A}_1 A_2) = 0{,}7\cdot(1-0{,}9) + (1-0{,}7)\cdot 0{,}9 = 0{,}34$$

$$P(\text{two exams passed}) = P(A_1 A_2) = 0{,}7\cdot 0{,}9) = 0{,}63$$

Law of distribution:

x_i	0	1	2
p_i	0,03	0,34	0,63

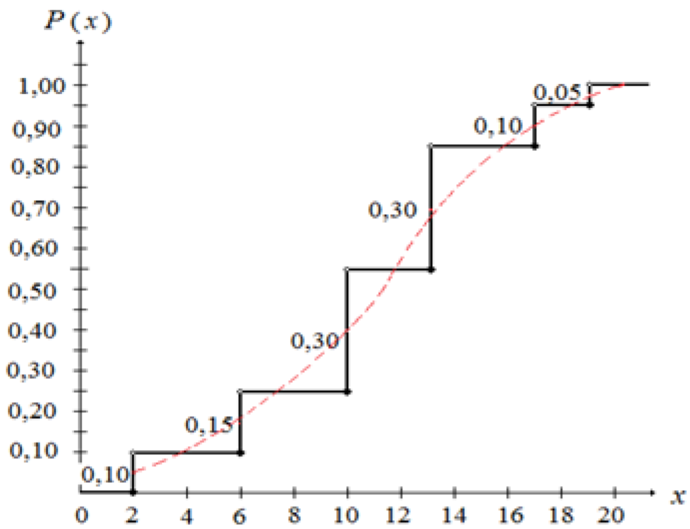
Function of distribution (integral function of distribution) of a random variable – that is a function which determines the probability that a random variable X takes a value less than some fixed real number x:

$$F(x) = P(X < x)$$

$$F(x) = P(X < x_i) = \sum_{x<x_i} p(x_i)$$

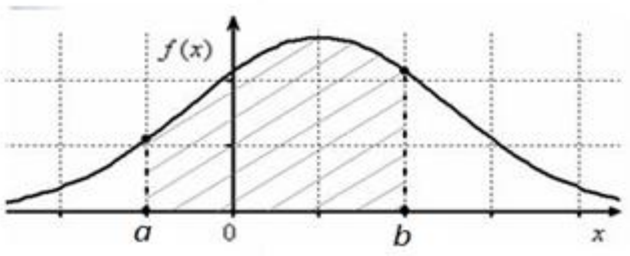
Example: Plot the graph of the function of distribution of a random variable X:

x_i – values of a random variable X	2	6	10	13	17	19	$X > 19$
p_i – probability of x_i	0, 10	0,15	0,30	0,30	0,10	0,05	0
$p(X < x_i)$	0	0,10	0,25	0,55	0,85	0,95	1,00



DENSITY OF PROBABILITY – that is the derivative of the integral function of distribution:

$$f(x) = F'(x)$$



$$P(a < X < b) = \int_a^b f(x)dx = F(b) - F(a)$$

NUMERICAL CHARACTERISTICS OF A RANDOM VARIABLE

Expected value of a discrete random variable *X* – that is the sum of the products of all possible values of the variable *X* into the probabilities of these values:

$$M(X) = \mu = x_1 p_1 + x_2 p_2 + ... + x_n p_n = \sum_{i=1}^{n} x_i p_i$$

For **continuous random variables** with a distribution density *f(x)*, the expected value (mathematical expectation) is equal to a definite integral:

$$M(X)=\int_a^b x\cdot f(x)dx$$

Variance, *D(X)* – that is mathematical expectation of the square of deviation of a random variable X from its mathematical expectation $M(X)$:

$$D(X)=M(X-M(X))^2$$

Root mean square deviation (r.m.s.d.), σ:

$$\sigma=\sqrt{D(X)}$$

Discrete random variable	Continuous random variable
$D(X)=\sum_{i=1}^{n}[x_i-M(X)]^2\cdot p_i$	$D(X)=\int_{-\infty}^{+\infty}[x-M(X)]^2\cdot f(x)dx$

Problem 2.3: A random variable X is given by the distribution law:

x_i	1	2	5
p_i	0,3	0,5	0,2

Determine $M(X)$, $D(X)$, σ.
Solution:
1) $M(X)$ = 1·0,3 + 2·0,5 + 5·0,2 = 2,3.
2)

x_i	1	2	5
p_i	0,3	0,5	0,2
$x_i - M(X)$	1 – 2,3	2 – 2,3	5 – 2,3
$(x_i - M(X))^2$	$(1 - 2,3)^2$	$(2 - 2,3)^2$	$(5 - 2,3)^2$

$$D(X)=M(X-M(X))^2=(1-2,3)^2\cdot 0,3+(2-2,3)^2\cdot 0,5+(5-2,3)^2\cdot 0,2=2,0$$

Root mean square deviation (r.m.s.d.), $\sigma=\sqrt{D(X)}=\sqrt{2,0}=1,42$

Problem 2.4: A random variable X is given by the distribution law:

x_i	2	5	6
p_i	0,2	0,5	0,3

Determine $M(X)$, $D(X)$, σ. Plot the graph of function of distribution.

Problem 2.5: A random variable X is given by the distribution law:

x_i	2	3	6	9
p_i	0,2	0,4	0,3	0,1

Determine $M(X)$, $D(X)$, σ. Plot the graph of function of distribution.

	Discrete random variable	Continuous random variable
Mathema-tical expectation (expected value)	$M(X)=\mu=\sum_{i=1}^{n} x_i \cdot p_i$	$M(X)=\mu=\int_{-\infty}^{+\infty} x \cdot f(x)dx$
Variance	$D(X)=\sigma^2=\sum_{i=1}^{n}[x_i-M(X)]^2 \cdot p_i =$ $= M(X^2)-[M(X)]^2$	$D(X)=\sigma^2=\int_{-\infty}^{+\infty}[x-M(X)]^2 \cdot f(x)dx =$ $= \int_{-\infty}^{+\infty} x^2 f(x)dx-[M(X)]^2 \cdot$

NORMAL LAW OF DISTRIBUTION

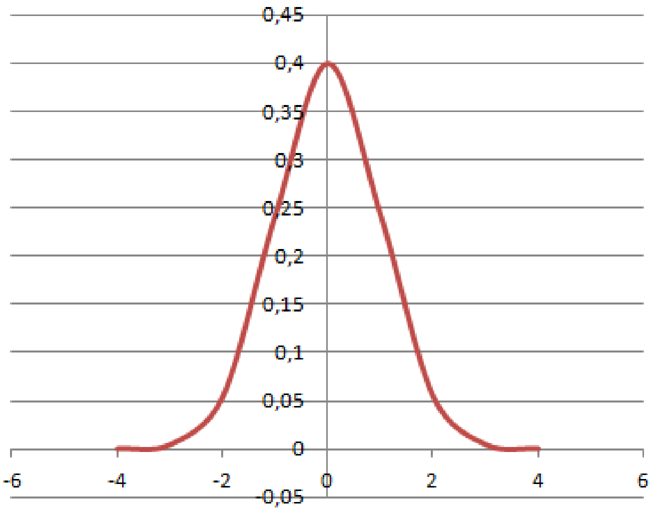
A continuous random variable distributed according to the normal law has a density function

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

and is uniquely determined by the parameters μ and σ (μ - mathematical expectation of a continuous random variable; σ - root mean square deviation)

Normal distribution with μ = 0 and σ = 1 is called standard (or normalized) normal distribution. It is denoted as ***N*(*x*, 0, 1). The density of probability of standard normal distribution is given by the formula:**

$$f(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$$

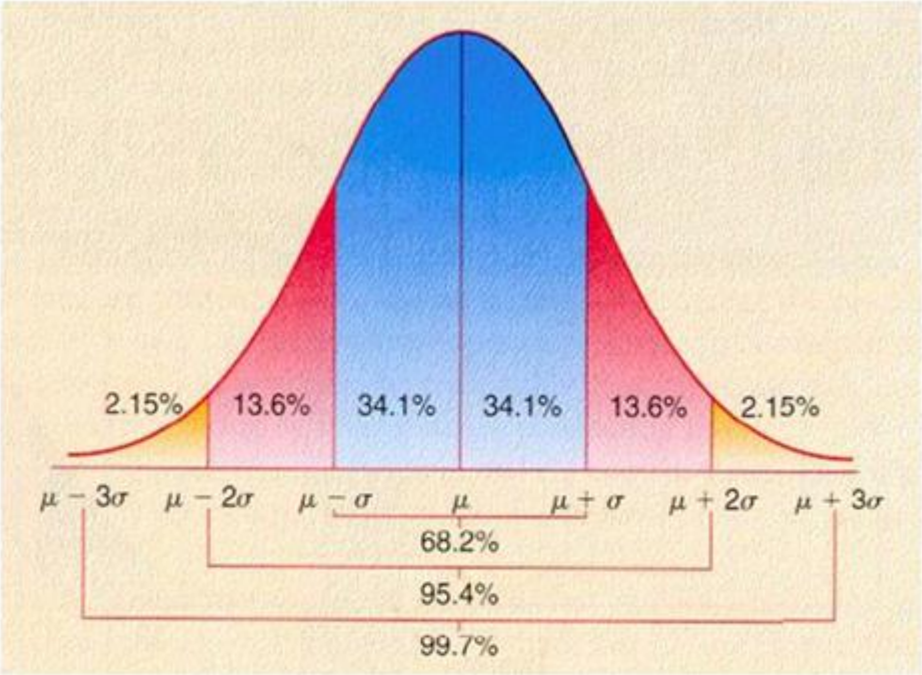


μ = 0; σ = 1

-6	-4	-3	-2	-1	0	1	2	3	4	6
6,08E-09	0,000134	0,004432	0,053991	0,241971	0,398942	0,241971	0,053991	0,004432	0,000134	6,08E-09

THREE SIGMA RULE

The essence of three sigma rule lies in the statement **that the fact that a normally distributed random variable takes value from the interval (μ - 3σ; $\mu + 3\sigma$) is practically reliable.**



Problem 2.6: What is the probability that a normally distributed random magnitude X takes a value within the interval $(\mu - \sigma, \mu + 2\sigma)$?

Problem 2.7: What is the probability that a normally distributed random magnitude X takes a value greater than $(\mu - 2\sigma)$?