Practical lesson No. 6.
Elements of Rheology and Hemodynamics

The student must learn the material on the topic and be able to answer the following questions:

1. Viscosity. Newton's formula. Newtonian and non-Newtonian fluids.
2. Blood as a non-Newtonian fluid. Influence of the properties of erythrocytes on the non-Newtonian character of blood.
3. Methods for determining the viscosity of fluids.
4. Laminar and turbulent flows. Reynolds number.
5. Stationary flow.
6. Poiseuille's formula.
7. Hydraulic resistance in series, parallel and combined piping systems. Branching vessels
8. Mechanical properties of some biological tissues.

Rheology - studies the processes of deformations and fluidity of substances (including the study of viscosity, elasticity and plasticity (inelastic deformation)). Rheological phenomena are associated with the molecular structure of substances.

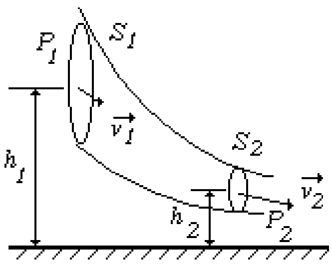
Hydrodynamics - section of physics studying the motion of fluids.

1. **Ideal fluid** - incompressible and non-viscous fluid.

Stationary flow of a fluid - the flow at which in a given point of fluid the velocity of flow remains constant with duration of time ($\vec{v} = const$).

The equation of continuity of a stream:

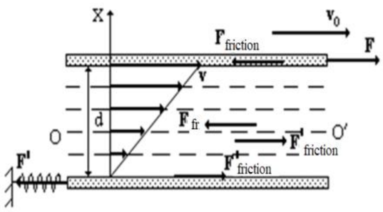
$v_1S_1 = v_2S_2$



Bernoulli's equation for the stationary flow of ideal fluid:

$$p_1 + \frac{\rho v_1^2}{2} + \rho g h_1 = p_2 + \frac{\rho v_2^2}{2} + \rho g h_2 = \text{const}$$

2. **Real fluids** - fluids in which there exists internal friction (viscosity) between the layers of fluid.



$$F_{fr} = \eta \frac{dv}{dx} S$$ -

Newton's equation for the force of internal friction

$\frac{dv}{dx}$ - shear rate (modulus of velocity gradient)

$$\text{SI}: \quad [\eta] = \frac{[F]}{\left[\frac{dv}{dx}\right][S]} = \frac{\text{N}}{\frac{\text{m}}{\text{s} \cdot \text{m}} \cdot \text{m}^2} = \text{Pa} \cdot \text{s}$$

$$\frac{F_{fr}}{S} = \sigma \quad - \quad \text{shear stress}$$

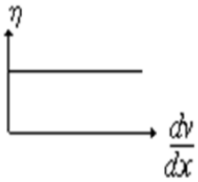
Shear stress is directly proportional to shear rate:

$$\sigma = \eta \cdot \text{grad } v$$

Newtonian fluid:

$$\eta = \eta(\text{temperature, kind of fluid}) \neq f\left(\frac{dv}{dx}\right)$$

η does not depend upon shear rate

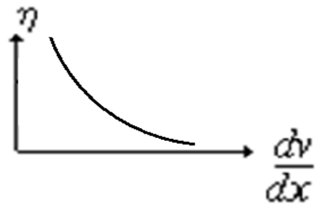


$$\boxed{F_{fr} = \eta \frac{dv}{dx} S}$$

Newtonian fluid

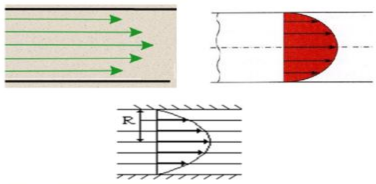
Non-Newtonian (anomalous) fluid:

$$\eta = \eta\left(\text{temperature, kind of fluid, } \frac{dv}{dx}\right).$$

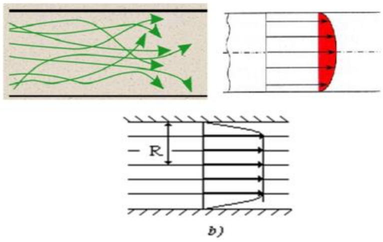


Blood is non-Newtonian fluid.

3. **Laminar flow -** the flow at which the layers of fluid are not mixed.



Turbulent flow - the flow at which the layers of fluid are mixed.



Reynolds number: $\mathrm{Re} = \frac{\rho v d}{\eta}$

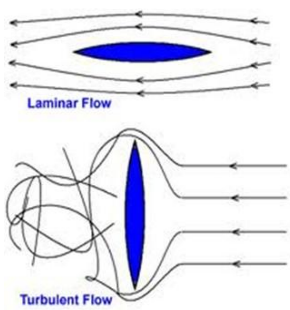
$$\mathrm{Re} = \frac{\rho v d}{\eta} = \frac{vd}{\eta/\rho} = \frac{vd}{\mu}, \qquad \text{with } \mu = \frac{\eta}{\rho} \text{ - kinematic viscosity}$$

If $\mathrm{Re} < \mathrm{Re}_{crit}$ => Laminar flow

If $\mathrm{Re} > \mathrm{Re}_{crit}$ => Turbulent flow

Reynolds number characterizes the relation between the forces of inertia and the forces of viscous friction:

$$\mathrm{Re} \propto \frac{\text{forces of inertia}}{\text{forces of viscous friction}}$$



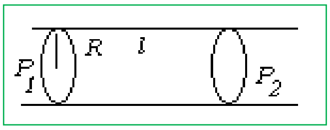
Water	Blood
$\mathrm{Re}_{crit} = 2300$	$\mathrm{Re}_{crit\ arteries} = 970 \pm 80$ $\mathrm{Re}_{crit\ capillaries} = 10^{-3} - 10^{-6}$

$Re_{crit\ arteries} >> 1 \quad \rightarrow \quad$ Viscosity << Forces of inertia
$Re_{crit\ capillaries} << 1 \quad \rightarrow \quad$ Viscosity >> Forces of inertia

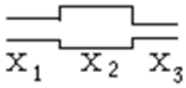
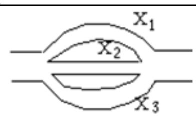
4. **Poiseuille's formula** allows to estimate the volumetric velocity Q of fluid flow:

$$Q = \frac{\Delta P}{X},$$

$$\frac{V}{t} = \frac{p_1 - p_2}{\dfrac{8\eta l}{\pi R^4}}$$

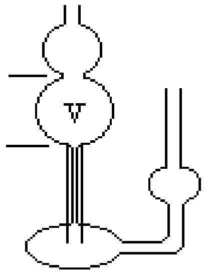


Hydraulic resistance $\quad X = \dfrac{8\eta l}{\pi R^4}$.

For the pipes connected in series	For the pipes connected in parallel
 $X_0 = X_1 + X_2 + X_3$	 $\dfrac{1}{X_0} = \dfrac{1}{X_1} + \dfrac{1}{X_2} + \dfrac{1}{X_3}$

Methods of determination of viscosity using Poiseuille's formula

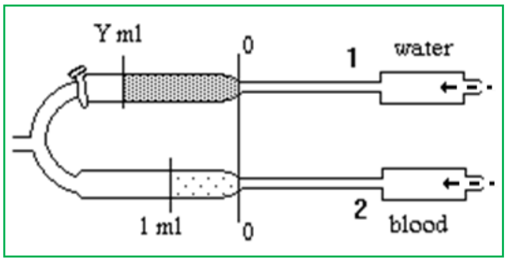
Ostwald viscometer: $\quad V = \dfrac{\Delta p}{X} \cdot t$



$$V = \frac{\rho_1 gh}{\dfrac{8\eta_1 l}{\pi R^4}} \cdot t_1 \quad ; \qquad V = \frac{\rho_2 gh}{\dfrac{8\eta_2 l}{\pi R^4}} \cdot t_2$$

$$\frac{\rho_1 gh}{\dfrac{8\eta_1 l}{\pi R^4}} \cdot t_1 = \frac{\rho_2 gh}{\dfrac{8\eta_2 l}{\pi R^4}} \cdot t_2 \qquad \Rightarrow \eta_2 = \eta_1 \frac{\rho_2 t_2}{\rho_1 t_1}$$

Hess viscometer is used to determine blood viscosity.
1) 1-capillary tube is filled with water till 0-mark, then the tap is closed
2) 2-capillary tube is filled with blood till 0-mark
3) the tap is opened
4) both fluids are sucked in, so that the blood comes to the mark "1 ml" and the water to the mark "Y ml".



For the volume of water: $Y = \dfrac{\Delta p}{\dfrac{8\eta_{water} l}{\pi R^4}} \cdot t$

for the volume of blood: $1 = \dfrac{\Delta p}{\dfrac{8\eta_{blood} l}{\pi R^4}} \cdot t \qquad \Rightarrow \quad \dfrac{Y}{1} = \dfrac{\eta_{blood}}{\eta_{water}} \quad \Rightarrow \quad \boxed{\eta_{blood} = Y \cdot \eta_{water}}$

Problem 6.1.

Observing the movement of red blood cells in the capillary under a microscope, it is possible to measure the blood flow rate ($v_{capillary}$ = 0.5 mm/s). The average blood flow velocity in the aorta is v_{aortha} =40 cm/s. Based on these data, determine how many times the sum of the cross sections of all functioning capillaries is greater than the cross section of the aorta.

Solution:

The equation of continuity of a stream: $v_1 S_1 = v_2 S_2$.

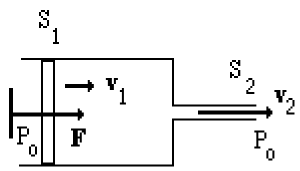
$$\frac{S_{capillaries}}{S_{aortha}} = \frac{v_{aortha}}{v_{capillaries}} = \frac{40 \cdot 10^{-2} \frac{m}{s}}{0{,}5 \cdot 10^{-3} \frac{m}{s}} = 800 .$$

Problem 6.2.

The velocity of water flow in a certain section of the horizontal pipe is v = 5 cm/s. Determine the flow velocity in that part of the pipe the diameter of which is two times smaller.

Problem 6.3.

From a horizontally located medical syringe with a diameter of d = 1.5 cm, a physiological solution is squeezed out with a force of F = 10 N. Find the speed of the solution flowing out of the syringe needle. The density of the saline solution is ρ = 1.03 g/cm^3. The piston cross section is much larger than the needle cross section.



Solution:

Bernoulli's equation for two sections of the syringe is: $p_0 + \frac{F}{S_1} + \frac{\rho v_1^2}{2} = p_0 + \frac{\rho v_2^2}{2}$,

where p_0 is atmospheric pressure, $\frac{F}{S_1}$ is the pressure created by the force acting on the piston. From here: $\frac{F}{S_1} = \frac{\rho v_2^2}{2} - \frac{\rho v_1^2}{2}$.

From the equation of continuity of a fluid flow $v_1 S_1 = v_2 S_2$ we have: $\frac{v_2}{v_1} = \frac{S_1}{S_2}$.

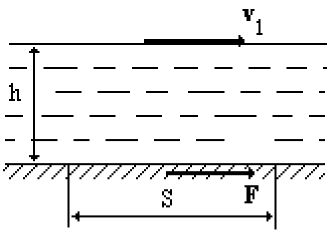
Since $S_1 >> S_2$, $v_2 >> v_1$. Then the member $\frac{\rho v_1^2}{2}$ may be neglected in comparison to $\frac{\rho v_2^2}{2}$. Then $\frac{F}{S_1} \approx \frac{\rho v_2^2}{2}$; from here $v_2 \approx \sqrt{\frac{2F}{\rho S_1}} = \sqrt{\frac{8F}{\rho \pi d^2}} = 10{,}5 \frac{\mathrm{m}}{\mathrm{s}}$.

Problem 6.4.

In the wide part of the horizontal pipe, water flows at a speed of v_1 = 50 cm/s. Determine the speed v_2 of water flow in the narrow part of the pipe, if the pressure difference in the wide and narrow parts Δp = 1.33 kPa.

Problem 6.5.

Calculate the tangential force on the area $S = 2$ m^2 of the river bottom, if the river depth $h = 2$ m. The velocity of the upper layer is $v_1 = 30$ cm/s, the velocity of the lower layers gradually decreases and is equal to zero at the bottom. At 20 ° C, the viscosity of water $\eta = 1005$ μPa · s



Solution:

Let’s determine the modulus of the velocity gradient and apply Newton's formula to calculate the friction force acting on the area S of the bottom:

$$\frac{dv}{dx} = \left|\frac{v_2 - v_1}{h}\right|; \qquad F_{\text{р}} = \eta \frac{dv}{dx} S = \eta \left|\frac{v_2 - v_1}{h}\right| S \approx 300 \text{ mcN}.$$

Problem 6.6.

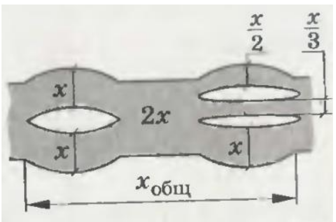
What is the hydraulic resistance of a blood vessel with a length of 0.12 m and a radius of 0.1 mm? Blood viscosity $\eta = 500$ μPa · s.

Solution:

According to the formula for hydraulic resistance, we get:

$$X = \frac{8\eta\, l}{\pi R^4} = \frac{8 \cdot 5000 \cdot 10^{-6}\,\text{Pa}\cdot\text{s}\cdot 0{,}12\text{m}}{3{,}14\cdot(10^{-4})^4\,\text{m}^4} \approx 150\cdot 10^{11}\,\frac{\text{Pa}\cdot\text{s}}{\text{m}^3}.$$

Problem 6.7. Determine the hydraulic resistance of a system of vessels



Solution:

1) $\dfrac{1}{X_1} = \dfrac{1}{x} + \dfrac{1}{x}$; $X_1 = \dfrac{x}{2}$.

2) $X_2 = 2x$

3) $\dfrac{1}{X_3} = \dfrac{1}{x/2} + \dfrac{1}{x/3} + \dfrac{1}{x} = \dfrac{6}{x}$; $X_3 = \dfrac{x}{6}$.

4) $X_{total} = X_1 + X_2 + X_3 = \dfrac{x}{2} + 2x + \dfrac{x}{6} = 2\dfrac{2}{3}x$.

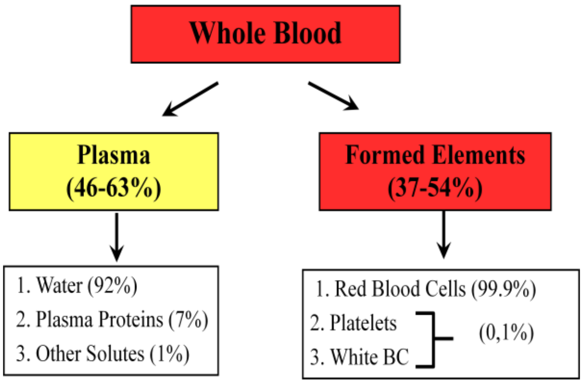
5. **Normal blood volume, o**n average, in men is 5.2 liters, in women - 3.9 liters, in newborns - 200-350 ml. The mass fraction of blood in the body of an adult is 6-8%.

Stroke volume (**SV**) is the volume of blood pumped from the left ventricle per beat. The term *stroke volume* can apply to each of the two ventricles of the heart, although it usually refers to the left ventricle. The stroke volumes for each ventricle are generally equal, both being approximately 60 - 70 ml in a healthy 70-kg man.

Cardiac output (**CO**), also known as **heart output** (denoted by the symbol *Q*) is the volume of blood being pumped by the heart per unit time (L/min). Cardiac output (CO) is the product of the heart rate (HR), i.e. the number of heartbeats per minute (bpm), and the stroke volume (SV):

$$\mathbf{CO = HR \times SV.}$$

For a healthy person weighing 70 kg, the **cardiac output** at rest averages about 5 L/min; assuming a heart rate of 70 beats/min, the stroke volume would be approximately 70 ml.



Blood consists of a liquid part (plasma, 52-60%) and formed elements (erythrocytes, leukocytes and platelets, 40-48%). (Blood is a suspension of a large number of blood cells in plasma).

Blood plasma is a water-salt protein solution of macromolecules (albumin (protein), lipids, carbohydrates)

Erythrocytes occupy about 45 ÷ 46% of the total blood volume. This value (0,46) is called the blood hematocrit.

Blood performs two major functions:

1) transports through the body:

- oxygen and carbon dioxide
- food molecules (glucose, lipids, amino acids)
- ions (e.g., Na^+, Ca^{2+}, HCO_3^-)
- wastes (e.g., urea)
- hormones
- heat

2) protects the body against infections and other foreign materials.

All the WBCs participate in these protection.

Commonly the following rheological properties of a fluid are considered:

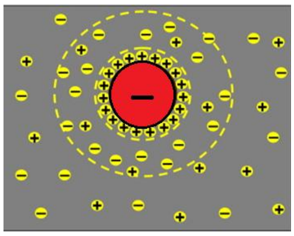
1) character of fluid (newtonian or non-newtonian); 2) density; 3) viscosity
4) critical Re

The **viscosity of human blood** is in the range from 4 to 6 mPa·s and in various pathologies varies from 1.5 to 23 mPa·s.

The value of blood viscosity can be used to diagnose health conditions and identify some diseases.

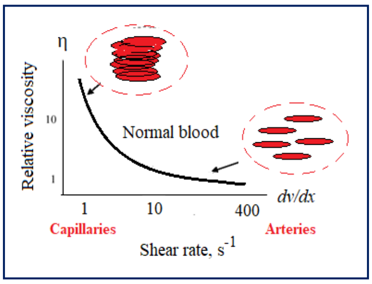
Water	**Blood**
newtonian ρ = 1000 kg/m^3 η = 1 mPa·s Re_{crit} = 2300	**non-newtonian (whole blood)** ρ = 1050 kg/m^3 η = 4-5 mPa·s $\eta_{\textbf{plazma=blood-blood cells}}$ = 1,2 mPa·s $\eta_{\textbf{serum=plazma-fibrinogen}}$=1,1 mPa·s $\eta_{\textbf{suspension of erythrocytes}}$=8-9 mPa·s $Re_{crit\ arteries}$ = 970 ± 80 $Re_{crit\ capillaries}$ = 10^{-3} - 10^{-6} **Plasma and serum – newtonian fluids**

The erythrocytes have the greatest influence on blood rheological properties.
The character of blood flow is influenced by:
- concentration of erythrocytes
- their physical properties.

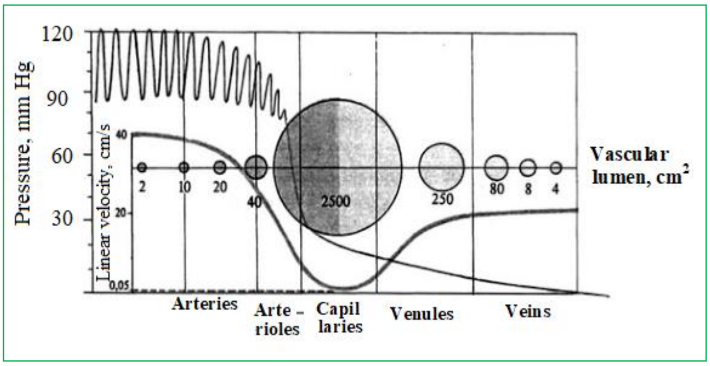


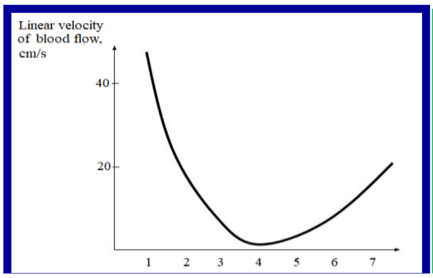
Main factors are:
1) shape (in pathologic state they can be spherical)
2) elasticity of membrane of erythrocyte (cholesterol makes it more rigid)
3) the ability to deform (when passing through capillaries the shape is changed)
4) existence of double electric layer around the erythrocyte
5) adhesion (they can attach to the walls of the vessel)
6) the ability to form columns (aggregates known as rouleaux) at low velocities of shear (due to fibrinogen).

The dependence of viscosity of blood upon the velocity of shear: elevated blood viscosity at low shear rates indicates RBC aggregation (rouleaux formation). Blood viscosity decreases with increasing shear rates as RBC aggregations breaks up to individual red cells.



Blood pressure, linear velocity of blood flow and vascular lumen in different sections of vascular system

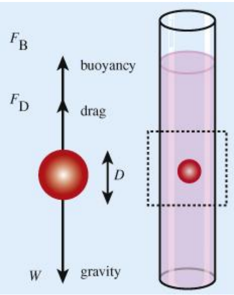




Linear blood flow velocity (v) in the vascular system:
1 - **aorta – 50 - 40 cm/s;**
2 - arteries – 40 - 20 cm/s;
3 - arterioles – 10 - 0.1 cm/s;
4 - **capillaries - 0.05 cm/s;**
5 - venules - 0.3 cm/s;
6 - veins - 0.3 - 5.0 cm/s;
7 - vena cava - 10.0 - 20.0 cm/s.

Problem 6.8.
How many times does the erythrocyte sedimentation rate change in people with spherocytosis compared to the norm, if the average erythrocyte radius in this disease increases by 1.5 times?
Solution:
Stoke’s formula for the force of viscous friction: $F_{fr} = 6\pi\eta Rv$

Buoyancy (Archimedes) force: $F_{Arch} = \rho_{serum} gV = \rho_{serum} g \frac{4}{3}\pi R^3$



Force of gravity: $mg = \rho_{erythr} Vg = \rho_{erythr} \frac{4}{3}\pi R^3 g$

When moving with constant velocity: $mg = F_{fr} + F_{Arch}$

$$\rho_{erythr} \frac{4}{3}\pi R^3 g = 6\pi\eta R v + \rho_{serum} g \frac{4}{3}\pi R^3 \text{, from here}$$

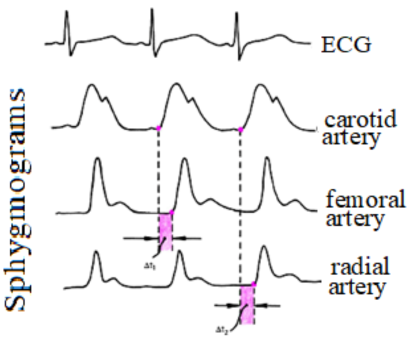
$$v = \frac{2}{9} \cdot \frac{(\rho_{erythr} - \rho_{serum}) g R^2}{\eta}.$$

For norm: $v_1 = \frac{2}{9} \cdot \frac{(\rho_{erythr} - \rho_{serum}) g R^2}{\eta}$.

In case of spherocytosis: $v_2 = \frac{2}{9} \cdot \frac{(\rho_{erythr} - \rho_{serum}) g (1{,}5R)^2}{\eta}$.

$\frac{v_2}{v_1} = (1{,}5)^2 = 2{,}25$.

6. **The wave of increased pressure which is caused by the throw-out of blood from the left ventricle during systole and which is spreading along aorta and the arteries is called the pulse wave**. In other words, the pulse wave is elastic oscillations of arterial walls which are caused by the wave of increased blood pressure, spreading along aorta and arteries.



Decremental pulse wave equation:

$$p = p_0 e^{-\chi x} \cos[\omega(t - \frac{x}{v})]$$

Pulse wave velocity (PWV) is about 5-10 m/s (that is a significant indicator of elastic properties of arterial vessels).

During the time of systole (0,3 s) the pulse wave will propagate at a distance of 1,5-3,0 m (greater than from the heart to the extremities). That means that the pulse wave front will reach the extremities earlier than the pressure in aorta starts to diminish. The pulsation of blood in big arteries will correspond to pulse wave, but the **velocity of blood flow (0,3 - 0,5 m/s, maximum) is rather smaller than PWV.**

PWV in big vessels depends upon their parameters as following:

$$\text{PWV} = v = \sqrt{\frac{Eh}{\rho d}} \quad \text{(Moens-Korteweg equation for PWV)},$$

where E - Young modulus of the vessel tissue, ρ - density of the vessel tissue; h - thickness of the vessel wall; d - diameter of the vessel.

When a person becomes older, E increases and VPWP increases.

Problem 6.9.

The speed of the pulse wave in the arteries is 8 m/s. What is the modulus of elasticity of these vessels if it is known that the ratio of the lumen radius to the vessel wall thickness is 6, and the density of the vascular wall is 1.15 g/cm^3?

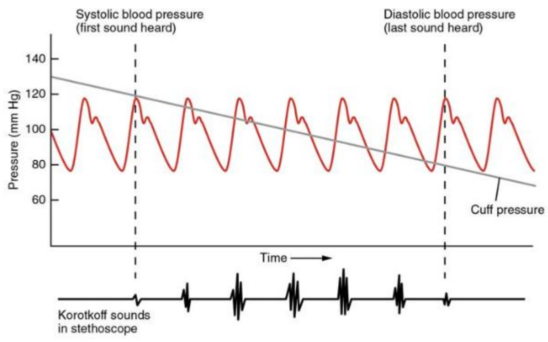
Solution:

$$v = \sqrt{\frac{Eh}{\rho d}}\ ;\ v^2 = \frac{Eh}{\rho d}\ ; E = \frac{v^2 \rho d}{h} = (8\frac{\text{m}}{\text{s}})^2 \cdot 1{,}15 \cdot 10^{-3}\frac{\text{kg}}{\text{m}^3} \cdot 2 \cdot 6 = 883\frac{\text{mN}}{\text{m}^2}$$

Problem 6.10.

How many times does the modulus of elasticity of the aortic wall change in atherosclerosis, if it is known that the pulse wave velocity has increased three times?

7. **Physical foundations of the clinical method for measuring blood pressure.** The sphygmomanometric method is based on measuring the external pressure required to compress the artery. When diminishing the external pressure turbulent flow occurs when the pressure in the cuff becomes equal to the systolic pressure. **Korotkoff sounds** are created by pulsatile blood flow through a compressed artery.



8. **Work** produced by the heart is spent to overcome the forces of pressure (A_1) and to give the blood kinetic energy (A_2):

$P_{mean} = 13$ kPa,
$V_{syst} = 60$ ml,
$\rho = 1050$ kg/m^3
$A_{heart\ at\ rest} \approx 1$ J
$t_{syst} \approx 0{,}3$ s .

$$A_1 = F \cdot l = P \cdot S \cdot l = P \cdot V_{syst}$$

$$A_2 = \frac{mv^2}{2} = \frac{\rho V_{syst} v^2}{2}$$

$$A_{leftventricle} = A_1 + A_2 = pV_{syst} + \frac{\rho V_{syst} v^2}{2}$$

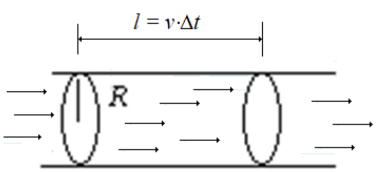
$$A_{rightventricle} = 0{,}2 A_{leftventricle}$$

$$A_{total} = A_{left} + A_{right} = 1{,}2(pV_{syst} + \frac{\rho V_{syst} v^2}{2})$$

$$P = \frac{A_{total}}{t} = \frac{1\text{J}}{0{,}3\text{s}} = 3{,}3\text{W}$$

Problem 6.11.

Determine the kinetic energy of the volume of blood flowing in one minute at a speed of 0.4 m/s through an artery with a diameter of 3 mm.

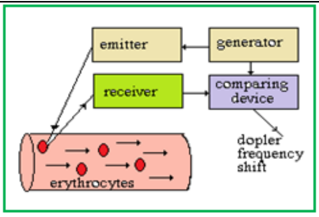
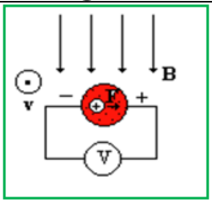


Solution:

$$W_{kin}=\frac{mv^2}{2}=\frac{\rho V\cdot v^2}{2}=\frac{\rho\cdot l\cdot S\cdot v^2}{2}=\frac{\rho\cdot v\cdot\Delta t\cdot\frac{\pi d^2}{4}\cdot v^2}{2}=\frac{\rho\cdot v^3\cdot\Delta t\cdot\pi d^2}{8}=$$

$$=\frac{1050\frac{\text{kg}}{\text{m}^3}\cdot(0{,}4\frac{\text{m}}{\text{s}})^3\cdot 60\text{s}\cdot 3{,}14\cdot(3\cdot 10^{-3}\text{m})^2}{8}\approx 0{,}014\text{ J}$$

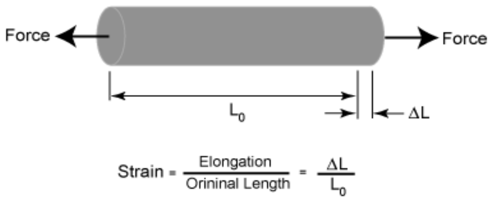
9. Determination of blood flow velocity

Ultrasonic method	**Electromagnetic method**
 $\Delta f\approx\frac{2v_{obj}}{v_{US}}f$	 $U=Ed=\frac{F}{q}d=\frac{qvB}{q}d=vBd$

10. MECHANICAL PROPERTIES OF SOLIDS AND BIOLOGICAL TISSUES

Hooke’s law: $\boxed{F=k\Delta l}$

Coefficient of elasticity: $F=E\cdot A\frac{\Delta l}{l}$

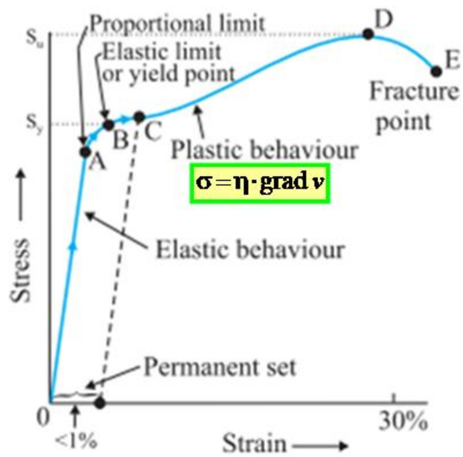


$\frac{F}{A}=\sigma$ (Stress); $\frac{\Delta l}{l}=\varepsilon$ (Strain)

Hooke’s law: $\boxed{\sigma=E\varepsilon}$

σ - mechanical stress; ε - the relative deformation (strain); A – cross section area; k – coefficient of elasticity; E – Young’s modulus (modulus of elasticity).

Stress-strain curve



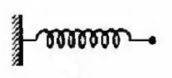
O – A; linear section, hence, Hooke’s Law obeys in this region.

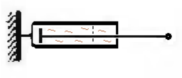
A – B; the stress and strain are not proportional. However, if the load is removed, the body returns to its original dimension. The point B in the curve is the Yield Point or the elastic limit and the corresponding stress is the Yield Strength (S_y) of the material.

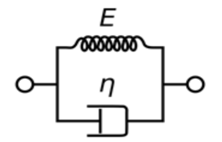
B - D; once the load is increased further, the stress starting exceeding the Yield Strength. This means that the strain increases rapidly even for a small change in the stress. This is shown in the region from B to D in the curve. If the load is removed at, say a point C between B and D, the body does not regain its original dimension. Hence, even

when the stress is zero, the strain is not zero and the deformation is called plastic deformation.
Further, the point D is the ultimate tensile strength (S_u) of the material. Hence, if any additional strain is produced beyond this point, a fracture can occur (point E).

Simulation of material (tissue)


Elastic element: $\sigma = E\varepsilon$


Viscous element: $F = r\dfrac{dx}{dt} \rightarrow \sigma = \eta\dfrac{d\varepsilon}{dt}$ (t – time of force action)



Kelvin-Voigt model

Elastic and viscous elements in parallel **(Kelvin-Voigt model)**

$$\varepsilon = \frac{\sigma}{E}(1 - e^{-\frac{Et}{\eta}})$$

Mechanical properties of biological tissues

Biological tissues are anisotropic (properties are different in different directions) composites (volumetric combination of dissimilar components).
Biological tissue is a composite of cellular and acellular material arranged in structures and substructures which properties can be altered during physiological growth and pathological events.
Collagen - triplehelix strong protein - is a fibrillar protein (high molecular weight compound), that forms the basis of the body's connective tissue (tendon, bone, cartilage, dermis, etc.) and provides its strength and elasticity. **Collagen is found in majority of tissues.**
Elastin is a connective tissue protein that has elasticity. Elastin performs important functions in organs subject to constant stretching and compression (in arteries, lungs, skin, tendons, various sphincters).

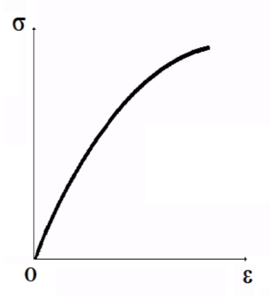
Tissue	Purpose, function	Components	Location
Collagenous fibers	Bind bones and other tissues to each other	Alpha polypeptide chains	tendon, ligament, skin, cornea, cartilage, bone, blood vessels, gut, and intervertebral disc
Elastic fibers	Allow organs like arteries and lungs to recoil	Elastic microfibril and elastin	extracellular matrix
Reticular fibers	Form a scaffolding for other cells		

Material	Elastic modulus, MPa	Ultimate strength, MPa
Collagen	10—100	100
Elastin	0,1—0,6	5

Bone is the main material of the musculoskeletal system. In a simplified form, it can be assumed that, bone mineral substance by **2/3 of the mass of compact bone tissue (0.5 volume) is inorganic material** hydroxylapatite $3Ca_3(PO4)_2{\bullet}Ca(OH)_2$. This substance is presented in the form of microscopic crystals. The rest of the bone (**1/3 of the mass**) consists of organic material, mainly **collagen.** Crystals of hydroxylapatite are located between collagen fibers (fibrils).

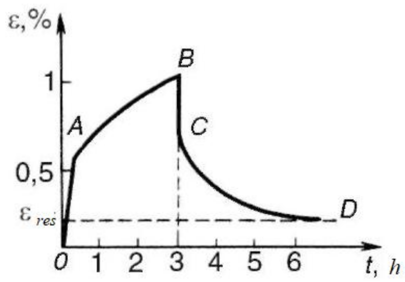
Bone density 2400 kg/m³. Its mechanical properties depend on many factors, including age, individual growth conditions of the organism and, of course, on the part of the organism.

The dependence $\sigma = f(\varepsilon)$ for compact bone tissue has a characteristic form, that is, it is similar to a similar dependence for a rigid body at small deformations, at which Hooke's law is fulfilled.

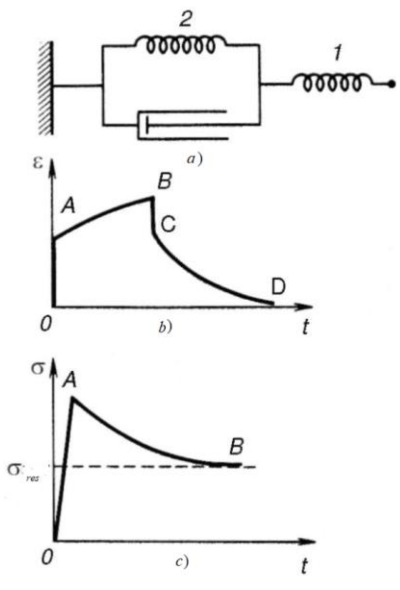


Young's modulus is about 10 GPa, ultimate strength is 100 MPa.

An approximate view of the creep curves of compact bone tissue. Section OA corresponds to rapid deformation, AB - to creep. At time t_1, corresponding to point B, the load was removed. BC corresponds to rapid deformation of contraction, CD - to reverse creep. As a result, even over a long period, the bone sample does not restore its previous size, and some residual deformation ε_{res} remains.



For this dependence, the following approximate model can be proposed (Fig. a). The time dependence of the relative deformation is shown in Fig. (b). Under the action of a constant load, spring 1 is instantly stretched (section OA), then the piston is extended (relaxation AB), after the termination of the load, spring 1 (BC) is rapidly compressed, and spring 2 retracts the piston into its previous position (reverse relaxation CD). The proposed model does not provide for residual deformation.



Schematically, it can be concluded that the mineral content of the bone provides rapid deformation, and the polymer part (collagen) determines creep.

If a permanent deformation is quickly created in the bone or in its mechanical model, then stress arises abruptly (section OA in Fig. c).

On the model, this means the extension of spring 1 and the occurrence of stress in it. Then (section AB) this spring will contract, pulling the piston and stretching the spring 2, the stress in the system will decrease. However, even after a considerable time, the residual stress will remain σ_{res}.

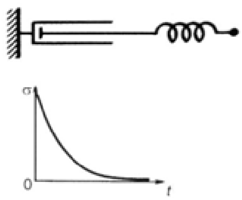
For the model, this means that under constant deformation there will be no such situation for the springs to return to undeformed states.

Skin consists of collagen fibers, elastin and the main tissue - the matrix. Collagen makes up about 75% of the dry matter and elastin makes up about 4%. Elastin stretches very strongly (up to 200-300%), approximately like rubber. Collagen can stretch up to 10%, which corresponds to nylon fiber. Skin is a viscoelastic material with highly elastic properties; it stretches and lengthens well.

Muscles

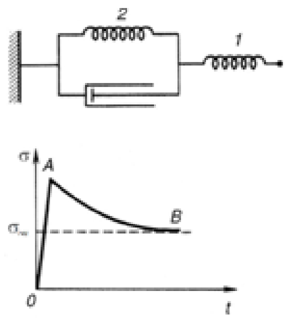
The muscles are composed of connective tissue, which consists of collagen and elastin fibers. Therefore, the mechanical properties of muscles are similar to those of polymers.

Relaxation of tension in smooth muscles corresponds to Maxwell's model (see Fig.). Therefore, smooth muscles can stretch significantly without much tension, which helps to increase the volume of hollow organs, such as the bladder.



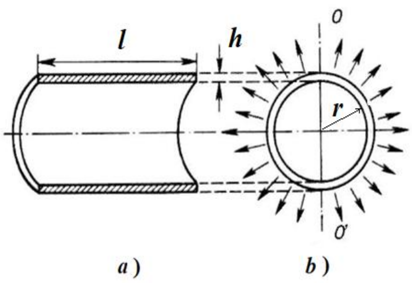
Simulation of a smooth muscle

The mechanical behavior of skeletal muscle corresponds to the model shown in Fig. With a rapid stretching of the muscles by a certain amount, the tension increases sharply, and then decreases to ε_{res}.



Simulation of skeletal muscle

The mechanical properties of **blood vessels** are mainly determined by the properties of **collagen, elastin and smooth muscle fibers**.



$$A = 2lh$$

$$F = \sigma A = \sigma 2lh$$

$$S = 2rl$$

$$p = \frac{F}{S} = \frac{\sigma 2lh}{2rl}$$

$$\boxed{\sigma = \frac{pr}{h}}$$ - Lame's equations for strain in the vascular wall.

Problem 6.12.

Determine the relative elongation of the skeletal muscle modeled by the Kelvin-Voigt body in 3 minutes, if the elastic modulus of the muscle is 1.2 MPa, the cross-sectional area is 0.8 $\cdot$ 10^{-6} m^2, and the load on the muscle is 6.3 N. Viscosity of muscle substance is equal to 1.25 g/(cm $\cdot$ s).

Solution:

$$\varepsilon = \frac{\sigma}{E}(1-e^{-\frac{Et}{\eta}}) = \frac{F}{A \cdot E}(1-e^{-\frac{Et}{\eta}}) = \frac{6{,}3\text{N}}{0{,}8 \cdot 10^{-6}\text{m}^2 \cdot 1{,}2 \cdot 10^6\text{Pa}}(1-e^{-\frac{1{,}2 \cdot 10^{-6} \cdot 3 \cdot 60}{0{,}125}}) = 6{,}5$$

Problem 6.13.

Determine the absolute elongation of the 5 cm tendon the diameter of which is 6 mm under the action of a force of 31.4 N. The elastic modulus of the tendon is 10^9 Pa.

Problem 6.14.

Determine the pressure in the wall of a capillary with a diameter of 20 μm, if the thickness of the vessel wall is 2 μm and the tangential stress in the wall is $8 \cdot 10^5$ Pa

Solution:

$$\sigma = \frac{pr}{h} = \frac{pd}{2h}; \quad p = \frac{2\sigma h}{d} = \frac{2 \cdot 8 \cdot 10^5\,\text{Pa} \cdot 2\ \mu\text{m}}{20\mu\text{m}} = 1{,}6 \cdot 10^5\ \text{Pa}$$