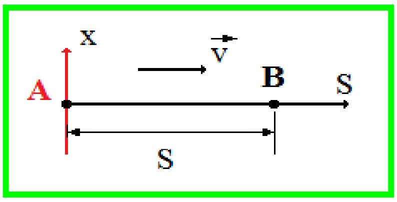
PRACTICAL LESSON No. 5
MECHANICAL WAVES. ACOUSTICS

Mechanical wave is the process of mechanical oscillations propagation in an elastic medium.
Wave surface - a surface formed by points oscillating in one and the same phase (at all points of which the wave has the same phase at a given time).
Wave front - the wave surface to which the wave has propagated at a given moment of time
Ray - normal to the wave front - shows the direction of wave propagation.

Transverse wave - the directions of oscillations of the particles of the medium and the propagation of the wave are mutually perpendicular. **Transverse waves can only be in solids.**
Longitudinal wave - the directions of oscillations of the particles of the medium and the propagation of the wave coincide. Longitudinal waves can occur in gases, liquids and solids.

HARMONIC OSCILLATIONS. PLANE WAVE EQUATION



$$x = x_0 \cdot \cos\omega(t - \frac{S}{v})$$

Wavelength $\lambda = v \cdot T$ $\quad v = \frac{\lambda}{T} = \lambda \cdot f$

The phase difference of oscillations of two points that are at a distance S from each other $\Delta\phi = \frac{2\pi}{\lambda} S$

Problem 5.1
Write the equation of harmonic vibration, if the acceleration amplitude a_{max} = 50 cm/s^2, the vibration frequency f = 0.5 Hz, the point displacement from the equilibrium position at the initial moment of time x_0 = 25 mm. Calculate the amplitude of the velocity.

a_{max} = 50 cm/s^2
f = 0.5 Hz
x_0 = 25 mm
$x = x(t)$ -?
v_{max} -?

Solution:
Equation of harmonic oscillations:
$x = x_{\max} \cdot \cos(\omega t + \varphi_0) = x_{\max} \cdot \cos(2\pi f \cdot t + \varphi_0)$;
Velocity:

$v = x' = (x_{max} \cdot \cos(\omega t + \varphi_0))' = -\omega x_{max} \cdot \sin(\omega t + \varphi_0)$;

Accelleration:

$a = v' = x'' = (-\omega x_{max} \cdot \sin(\omega t + \varphi_0))' = -\omega^2 x_{max} \cdot \cos(\omega t + \varphi_0)$;

$a_{max} = \omega^2 x_{max}$.

Amplitude of oscillations: $x_{max} = \frac{a_{max}}{\omega^2} = \frac{a_{max}}{(2\pi f)^2} = \frac{0{,}5\,\frac{\text{m}}{\text{s}^2}}{(2 \cdot 3{,}14 \cdot 0{,}5\text{Hz})^2} = 0{,}05 \text{ m}$.

$x = x_{max} \cdot \cos(\omega t + \varphi_0)$

At $t = 0$ the displacement $x = x_0$

$x_0 = x_{max} \cdot \cos(\omega \cdot 0 + \varphi_0)$

$0{,}025 = 0{,}05 \cdot \cos\varphi_0$

$\cos\varphi_0 = 0{,}5$; $\varphi_0 = \frac{\pi}{3}$

Equation of harmonic oscillations:

$$x = x_{max} \cdot \cos(\omega t + \varphi_0) = x_{max} \cdot \cos(2\pi f \cdot t + \varphi_0) = 0{,}05 \cdot \cos(2\pi \cdot 0{,}5 \cdot t + \frac{\pi}{3}) =$$

$$= 0{,}05 \cdot \cos(\pi \cdot t + \frac{\pi}{3}).$$

Amplitude of the velocity:

$$v_{max} = \omega x_{max} = 2\pi f \cdot x_{max} = 2 \cdot 3{,}14 \cdot 0{,}5\text{Hz} \cdot 0{,}05\text{m} = 0{,}16\,\frac{\text{m}}{\text{s}}.$$

Problem 5.2

Write the equation of harmonic oscillations, if the amplitude of the velocity v_{max} = 63 cm/s, the period of oscillations T = 1 s, the displacement of the point from the equilibrium position at the initial moment of time is equal to zero. Determine the amplitude of acceleration, the frequency of oscillations.

Problem 5.3

The sound source vibrates according to the law $x = \sin 2000\pi t$. Sound propagation speed 340 m/s. Write the equation of vibrations for a point located at a distance of 102 m from the source. Energy loss can be neglected. The wave is considered flat.

Problem 5.4

Calculate the phase difference of oscillations of two points lying on the same ray and spaced from each other at a distance of S = 1.75 m, if the wavelength λ = 1 m.

S = 1.75 m
λ = 1 m
$\Delta\varphi$ -?

Solution:

$$\Delta\varphi = \frac{2\pi}{\lambda} S = \frac{2\pi}{1\text{ m}} 1{,}75 \text{ m} = 3{,}5\pi$$

Problem 5.5.
What is the vibration frequency if the smallest distance between points vibrating in the same phases is S = 1 m? Wave propagation speed is 300 m/s.

Problem 5.6.
Compare the phase difference in the pulse wave between two points located 20 cm apart. Consider the speed of the pulse wave equal to 10 m/s, and the heart oscillations - harmonic with a frequency of 1.2 Hz

ENERGY TRANSFER

Volumetric density of energy = Energy per unit volume $w_\rho = \frac{\rho\omega^2 x_0^2}{2}$

Energy flux is the ratio of the energy carried by the wave through the surface to the transfer time: $\Phi = \frac{W}{\Delta t}$ $\left(\frac{\mathrm{J}}{\mathrm{s}}\right)$

Wave intensity (energy flux density) is the ratio of the energy flux to the area perpendicular to the direction of wave propagation

$$I = \frac{\Phi}{S} = \frac{W}{\Delta t \cdot S} \qquad \left(\frac{\mathrm{J}}{\mathrm{s \cdot m^2}} = \frac{\mathrm{W}}{\mathrm{m^2}}\right)$$

$$I = w_\rho \cdot c = \frac{\rho\omega^2 x_0^2}{2} \cdot c;$$

$$I = \frac{P^2}{2\rho c}.$$

Specific Wave Resistance (acoustic impedance, *Z*)

$$Z = \frac{P}{v},$$

where P – sound pressure;
v – **velocity of oscillating particles** of the medium.
Specific wave resistance (acoustic impedance, *Z*) depends directly proportional upon the product of the medium density (ρ) and the speed of sound (c) in this medium:

$$Z = \rho \cdot c$$
$$P = Z \cdot v = \rho \cdot c \cdot v$$

Coefficient of the sound wave penetration $\beta = \frac{I_2}{I_1}$, where

I_1 – incident wave intensity, I_2 – penetrating wave intensity

$$\beta = 4\frac{\frac{c_1\rho_1}{c_2\rho_2}}{\left(\frac{c_1\rho_1}{c_2\rho_2}+1\right)^2}.$$

If $c_1\rho_1 << c_2\rho_2$, then $\beta \approx 4\frac{c_1\rho_1}{c_2\rho_2}$.

Problem 5.7.
The intensity of a plane wave in air is 10^{-10} W/m^2. Determine the vibration amplitude of air particles (molecules) under normal conditions (sound speed in air 330 m/s; air density 1,29 kg/m^3) and the volumetric density of energy of vibrational motion a) for frequency of f_1 = 20 Hz.
Solution:

$$I = w_\rho \cdot c = \frac{\rho\omega^2 x_0^2}{2}\cdot c; \quad w_\rho = \frac{I}{c} = \frac{10^{-10}\ \text{W/m}^2}{330\ \text{m/c}} = 3\cdot 10^{-13}\frac{\text{J}}{\text{m}^3}.$$

$$w_\rho = \frac{\rho\omega^2 x_0^2}{2};\ x_0 = \sqrt{\frac{2w_\rho}{\rho\omega^2}} = \sqrt{\frac{2\cdot 3\cdot 10^{-13}\frac{\text{J}}{\text{m}^3}}{1{,}29\frac{\text{kg}}{\text{m}^3}\cdot(2\cdot 3{,}14\cdot 20)^2\frac{1}{\text{s}^2}}} = 5\ \text{nm}.$$

Determine the vibration amplitude of air particles (molecules) for frequencies of
b) f_2 = 1000 Hz,
c) f_3 = 20000 Hz.

Problem 5.8. The study of the movement of the tympanic membrane showed that the speed of vibration of its sections is of the same order of magnitude as the speed of displacement of air molecules during the propagation of a plane wave. Based on this, calculate the approximate amplitude of oscillations of the eardrum

a) for the threshold of hearing ($I = 10^{-12}\frac{\text{W}}{\text{m}^2}$).

Solution: $v \approx v_{\max} = \omega x_{\max};\ x_{\max} = \frac{v}{\omega}$.

$$P = Z\cdot v = \rho\cdot c\cdot v$$

$$I = \frac{P^2}{2\rho c} = \frac{P^2}{2Z}; \quad P = \sqrt{2ZI}$$

$$v = \frac{P}{Z} = \frac{\sqrt{2ZI}}{Z} = \sqrt{\frac{2I}{Z}} = \sqrt{\frac{2I}{\rho c}};$$

$$x_{\max} = \frac{v}{\omega} = \frac{1}{2\pi f}\sqrt{\frac{2I}{\rho c}} = \frac{1}{2\cdot 3{,}14\cdot 1000\ \text{Hz}}\sqrt{\frac{2\cdot 10^{-12}\frac{\text{W}}{\text{m}^2}}{1{,}29\frac{\text{kg}}{\text{m}^3}\cdot 330\frac{\text{m}}{\text{s}}}} = 0{,}01\ \text{nm}$$

b) for the threshold of pain $(I = 10 \dfrac{\text{W}}{\text{m}^2})$.

Problem 5.9.
Determine the average force acting on the eardrum (area A = 66 mm^2)

a) for the threshold of hearing $(I = 10^{-12} \dfrac{\text{W}}{\text{m}^2})$.

Solution:

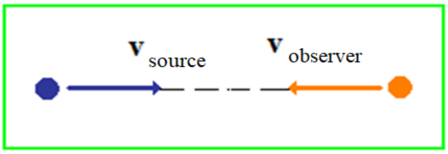
$$I = \frac{P^2}{2\rho c}; \quad P = \sqrt{2\rho c I}$$

$$F = P \cdot A = \sqrt{2\rho c I} \cdot A = \sqrt{2 \cdot 1{,}29 \frac{\text{kg}}{\text{m}^3} \cdot 330 \frac{\text{m}}{\text{s}} \cdot 10^{-12} \frac{\text{W}}{\text{m}^2}} \cdot 66 \cdot 10^{-6}\ \text{m}^2 = 1{,}93\ \text{nN};$$

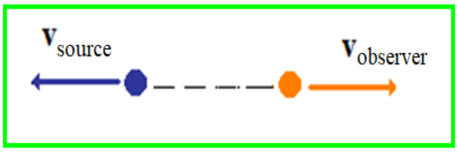
b) for the threshold of pain $(I = 10 \dfrac{\text{W}}{\text{m}^2})$.

THE DOPPLER EFFECT

The Doppler effect is a change in the frequency of wave oscillations recorded by the observer (receiver) when the source (S) of the wave and the observer (O) move relative to each other.



$$f'' = \frac{v_{\text{sound}} + v_{\text{obs}}}{v_{\text{sound}} - v_{\text{source}}} f$$



$$f'' = \frac{v_{\text{sound}} - v_{\text{obs}}}{v_{\text{sound}} + v_{\text{source}}} f$$

In medical applications $\Delta f \approx \dfrac{2v_{obj}}{v_{US}} f$.

Problem 5.10.
Two cars are moving towards each other at speeds of v_1 = 20 m/s and v_2 = 10 m/s. The first car gives a signal with a frequency of 800 Hz. What signal frequency will the driver of the second car hear

a) before the cars meet

$$f'' = \frac{v_{\text{sound}} + v_{\text{obs}}}{v_{\text{sound}} - v_{\text{source}}} f = \frac{330 + 10}{330 - 20} \cdot 800 = 877\ \text{Hz}$$

b) after the cars meet

Problem 5.11.
The Doppler frequency shift when a mechanical wave is reflected from moving erythrocytes is 50 Hz, the generator frequency is 100 kHz. Determine the velocity of blood movement in a blood vessel.
Solution:

$$\Delta f \approx \frac{2v_{erihr}}{v_{US}} f\,; \quad v_{erithr} = \frac{\Delta f \cdot v_{US}}{2f} = \frac{50\ \text{Hz} \cdot 1540\ \text{m/s}}{2 \cdot 100 \cdot 10^3\ \text{Hz}} = 0{,}39\ \text{m/s}$$

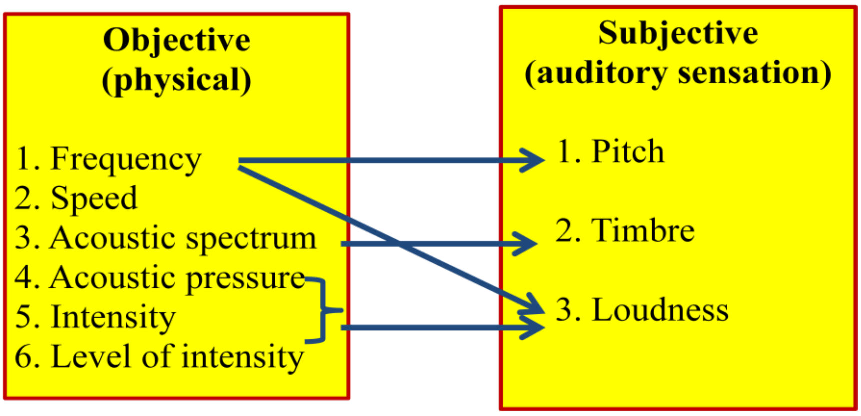
Problem 5.12.
By what percentage will the frequency of ultrasound change when it is reflected from moving red blood cells in an artery? Take the average speed of movement of erythrocytes equal to 40 cm/s.

Sound waves (audible sound) - longitudinal mechanical waves with vibrations from 16 Hz to 20 kHz, which propagate in an elastic medium and are perceived by the human ear.

Infrasound $f < 16$ Hz	Sound 16 Hz $< f <$ 20 kHz	Ultrasound 20 kHz $< f <$ 1 GHz

Acoustic spectrum - a set of frequencies with an indication of their relative intensity (amplitude). Spectrum is shown as a graph of amplitude versus frequency.

Characteristics of sound



Speed of sound in air 330 m/s (340 m/s at 20°C), in water - 1500 m/s, in blood – 1540 (1570) m/s.

Sound intensity $I = w_\rho \cdot c = \frac{\rho\omega^2 x_0^2}{2} \cdot c\,; \quad I = \frac{P^2}{2\rho c}$

Hearing threshold (audibility threshold) at 1 kHz $I_0 = 10^{-12}\ \frac{\text{W}}{\text{m}^2}$

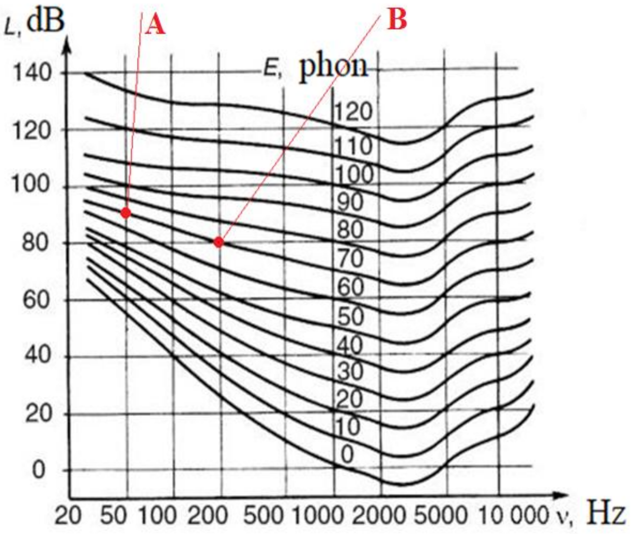
Pain threshold (boundary) $I_{pain} = 10\ \frac{\text{W}}{\text{m}^2}$

Level of intensity $L = \lg \frac{I}{I_0}$ [B] $L = 10 \lg \frac{I}{I_0}$ [dB]

To compare **loudness (auditory sensation)** of different sound the scale of level of loudness (E) is used. Unit for loudness level is **phon**.

At f = 1 kHz 1 phon = 1 dB ;

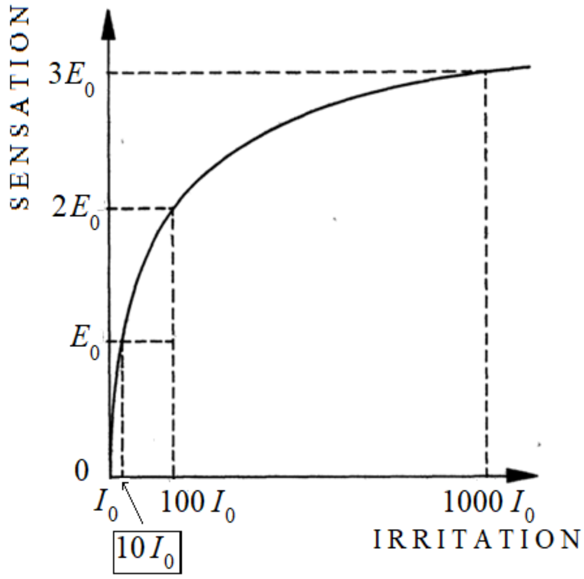
$$L = E = 10 \lg \frac{I}{I_0}.$$



Sound "A": L = 90 dB, f = 50 Hz
Sound "B": L = 80 dB, f = 200 Hz
Sounds "A" and "B" produce the sensation of one and the same loudness of 70 phons.

Sound intensity level L (dB)	Intensity I (W/m^2)	Example/effect
0	1×10^{-12}	Threshold of hearing at 1000 Hz
10	1×10^{-11}	Rustle of leaves
20	1×10^{-10}	Whisper at 1 m distance
30	1×10^{-9}	Quiet home
40	1×10^{-8}	Average home
50	1×10^{-7}	Average office, soft music
60	1×10^{-6}	Normal conversation
70	1×10^{-5}	Noisy office, busy traffic
80	1×10^{-4}	Loud radio, classroom lecture
90	1×10^{-3}	Inside a heavy truck; damage from prolonged exposure
100	1×10^{-2}	Noisy factory, siren at 30 m; damage from 8 h per day exposure
110	1×10^{-1}	Damage from 30 min per day exposure
120	1	Loud rock concert, pneumatic chipper at 2 m; threshold of pain
140	1×10^{2}	Jet airplane at 30 m; severe pain, damage in seconds
160	1×10^{4}	Bursting of eardrums

Weber-Fechner psychophysical law: sensation of irritation is proportional to the logarithm of the intensity of irritation. The sensation grows in an arithmetic progression if the intensity of the stimulus increases in geometric progression (exponentially).



Irritation (Intensity, I)	I_0	aI_0	a^2I_0	a^3I_0
Sensation (Loudness, E)	0	E_0	$2E_0$	$3E_0$

Problem 5.12.
Two sounds of the same frequency differ in intensity by ΔL = 30 dB. Determine the ratio of the sound pressure amplitudes.
Solution:

$$L_1 = 10\lg\frac{I_1}{I_0} = 10\lg\frac{\frac{P_1^2}{2\rho c}}{\frac{P_0^2}{2\rho c}} = 10\lg\left(\frac{P_1}{P_0}\right)^2 = 20\lg\frac{P_1}{P_0}$$

$$L_2 = 10\lg\frac{I_2}{I_0} = 20\lg\frac{P_2}{P_0}.$$

$$\Delta L = L_2 - L_1 = 20\lg\frac{P_2}{P_0} - 20\lg\frac{P_1}{P_0} = 20\lg\frac{P_2}{P_1}$$

$$30 = 20\lg\frac{P_2}{P_1}; \qquad \frac{P_2}{P_1} = 10^{\frac{3}{2}} = 32$$

Problem 5.13.
Sound with a frequency of f = 200 Hz travels a certain distance in an absorbing medium. In this case, the sound intensity decreases from $I_1 = 10^{-4}$ W/m^2 to $I_2 = 10^{-8}$ W/m^2. What is the diminution of loudness level?
Solution:

$$L_1 = 10\lg\frac{I_1}{I_0} = 10\lg\frac{10^{-4}}{10^{-12}} = 80 \text{ дБ}$$

$$L_2 = 10\lg\frac{I_2}{I_0} = 10\lg\frac{10^{-8}}{10^{-12}} = 40 \text{ dB}$$

By curves of equal loudness at 200 Hz $L_1 = 80$ dB corresponds to $E_1 = 70$ phons; $L_2 = 40$ dB corresponds to $E_2 = 20$ phons; the change (diminution) of loudness level is 50 phons.

Problem 5.14.
Noise on a busy street corresponds to the loudness level E_1 = 70 phon, scream to E_2 = 80 phon. What will be the loudness level of the sound resulting from the addition of shouting and street noise? Consider the sound frequency *f* to be equal to 1 kHz.

PHYSICAL FOUNDATIONS OF SONIC RESEARCH METHODS IN CLINICS

Percussion - a method for examining internal organs, based on tapping on the surface of the patient's body with an assessment of the nature of the sounds that arise during this.

Auscultation – method of internal organs research, based on listening to sound phenomena that arise during the physiological activity of internal organs.

US-examination

Ultrasound (US) - mechanical vibrations and waves, the frequency of which is more than 20 kHz.

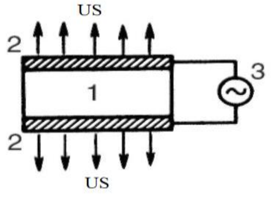
The upper limit of ultrasonic frequencies (10^9 -10^{10} Hz) is determined by intermolecular distances and therefore depends on aggregate state of the substance in which the ultrasonic wave propagates.

US waves:

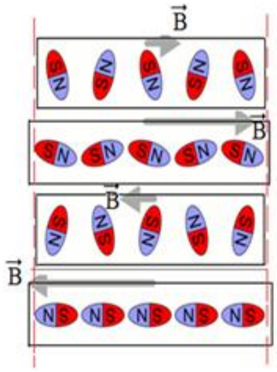
- are of ray character
- Obey the laws of reflection and refraction
- Easy focused
- high intensities can be obtained

GENERATION OF US WAVES

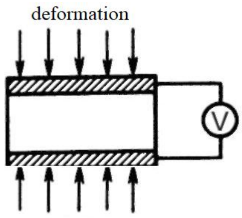
Inverse piezoelectric effect - mechanical deformation of bodies under the action of an electric field, *f* = 10 MHz



Magnetostriction – vibration of ferromagnetic rod (Fe, Ni) in an alternating magnetic field, $f = 50$ kHz



Receiver of US wave (direct piezoelectric effect)



The effect of ultrasound on cells and tissues of the body	
Mechanical	**Physical and chemical**
- Micro-massage of cells and tissues - Reconstruction of BM - Destruction of biomacromolecules - Destruction of cells and microorganisms - Change in BM permeability	- Ionization and dissociation of molecules of substance and formation of biologically active molecules - increased enzyme activity

ULTRASONIC WAVES APPLICATIONS IN MEDICINE		
Diagnostics	**Treatment**	
Echolocation methods (US reflection) $I = 50$ mW/cm^2 $f = 1 – 30$ MHz (Commonly 2,25 - 5 MHz) Since at the border of the transition of ultrasound from air to the skin, 99.99% of the vibrations are reflected, then during ultrasound scanning of the patient it is necessary to **lubricate the skin surface with water jelly, which acts as a transition medium.**	**Physiotherapy** (Low intensities) $f = 880$ kHz $I = 1$ W/cm^2 Depth of penetration 3-5 cm Ultrasonic inhalation Phonophoresis **Ultrasound acupuncture**	**Ultrasound surgery** High intensities $I = 10^3$ W/cm^2 Purpose: to induce controlled selective destruction in tissues. Two methods: - Deterioration of tissues by ultrasound $f = 4$ MHz - Reduced cutting force $f = 50$ kHz **Ultrasonic scalpel HARMONIC** **US osteosynthesis**

Modes of scanning

A-mode (Amplitude mode)

The technique provides information in the form of a one-dimensional image (vertical, Y-axis - the amplitude of the reflected signal from the boundary of media with different acoustic impedance, X-axis - is the distance to this boundary. **Used to scan static objects.**

B-Mode (Brightness) is a two-dimensional ultrasound image display composed of bright dots representing the ultrasound echoes.

The brightness of each dot is determined by the amplitude of the returned echo signal. This allows for visualization and quantification of anatomical structures, as well as for the visualization of diagnostic and therapeutic procedures for small animal studies.

This universal imaging mode is great for **image-guided injections** for needle placement of an injection or aspiration procedure, identification of lesions, cysts or tumors, locating structural anomalies, visualizing cardiac and vascular movement across the cardiac cycle.

M-mode (Motion mode)

The technique gives information in the form of a one-dimensional image, the second coordinate is replaced by a time one. The vertical axis is the distance from the sensor to the structure to be located, and the horizontal is the time. The mode is used mainly for examining the heart. Gives information about the shape of the curves reflecting the amplitude and speed of movement of cardiac structures.